# Isospin in Be8 \*

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Dedicated to Professor Dr. W. Gentner on the occasion of his 60th birthday

Many experiments indicate that isospin is not a good quantum number for the states with  $J^{\pi}=2^{+}$  at 16.6 MeV and 16.9 MeV in Be<sup>8</sup>. The available experimental information is examined and compared with a model of single particle states which are maximally mixed in isospin, as proposed by Marion. An analysis of all data pertinent to the states at 17.6 MeV and 18.15 MeV with  $J^{\pi}=1^+$ shows that these states are rather pure (90%) isospin states. The state at 17.6 MeV has T=1, the one at 18.15 MeV hat T=0. Both the  $2^+$  and the  $1^+$  levels form doublets with similar wave functions in each case. A similar pair is most likely formed by the 3+ states at 19.05 MeV and 19.22 MeV, as proposed by Barker. Reduced widths indicate possible isospin mixing of as much as 30% for these states. Some experimental evidence is presented for another pair of states with  $J^{\pi}=1^{-}$  at 20.35 MeV and 22 MeV, and with  $J^{\pi}=2^{-}$  at 18.9 MeV and 23.6 MeV. This hypothesis provides analog states in Be8 for every state observed in Li8 in the energy region which is considered. If each pair (except the 1<sup>+</sup> and 2<sup>+</sup> pairs) is allowed to mix isospin by at most 30%, all cases of large excess of either neutron or proton reduced width can be explained. The large isospin mixture in the 2+ pair can be explained on the basis of an initial energy degeneracy of these states (removed by the COULOMB force). The intermediate coupling shell model does account for such an accident. It is argued that, although a simple cluster picture of these states is highly successful in a qualitative way, the intermediate coupling model is able and probably needed to achieve a quantitative description of Be8 between 16 and 20 MeV.

While the level scheme of Be<sup>8</sup> exhibitis only the rotational level sequence 0<sup>+</sup>, 2<sup>+</sup>, 4<sup>+</sup> at low energies <sup>1</sup>, it becomes complex at about 16 MeV. This is qualitatively understood in a cluster picture. The rotational sequence arises from the two-alpha-particle structure of the Be8 ground state. Only the breakup of an alpha cluster can produce more complex states, and this requires an energy of about the (Li<sup>7</sup>+p) separation energy (about 17 MeV). Since two alpha particles cannot form states with isospin T=1, the first states of such character will appear in the region of these single particle cluster states. More specifically, the first state with T=1 should lie between 16 MeV and 18 MeV, as can be inferred from the position of the ground states of Li<sup>8</sup> and B<sup>8</sup>. The location and electromagnetic decay properties of the analog state to the Li8 - B8 ground states are of great interest <sup>2</sup> because the A = 8 triad has been proposed as a test case for the validity of the Conserved Vector Current Theory for beta decay. While a similar comparison was made successfully in C12, where the first T=1 level at 15.11 MeV is well known and extremely pure in its isospin character, in Be8 the decay of such a state has only been estimated from theoretical calculations <sup>2</sup>. In the search for this state, a number of curious facts were discovered which suggested extreme isospin mixing <sup>3, 4</sup> in some states of Be<sup>8</sup>. This is a very interesting hypothesis because in light nuclei isospin has, so far, proven to be a good quantum number <sup>5</sup> to within 10% (this refers to the squares of the amplitudes).

This article presents a critical survey of the available experimental evidence for such mixing, and discusses some of the specific models proposed to explain the effects. For the case of the states at 16.6 MeV and 16.9 MeV, most of the experimental data have been compiled by Marion and Wilson 6. The present communication extends this to the higher states in Be<sup>8</sup> and, in particular, the analog states of the first excited state in Li<sup>8</sup>.

## I. Level Assignments in Li<sup>8</sup> and Be<sup>8</sup>

States with isospin T=1 in Be<sup>8</sup> can be located by directly identifying the isospin of such states experimentally. As this is mostly not possible one

- \* Work supported in part by the National Science Foundation.
- <sup>1</sup> T. Lauritsen and F. Ajzenberg-Selove, Energy Levels in Light Nuclei VII, Nucl. Phys. 78, 1 [1966].
- <sup>2</sup> H. A. Weidenmuller, Phys. Rev. Letters 4, 299 [1960].
- <sup>3</sup> J. B. Marion, Physics Letters 14, 315 [1965].
- <sup>4</sup> D. Kohler and P. Paul, Physics Letters 15, 157 [1965].
- W. M. McDonald, Nuclear Spectroscopy B (editor F. Ajzen-BERG-SELOVE), Academic Press 1960.
- <sup>6</sup> J. B. Marion and M. Wilson, Nucl. Phys. 77, 129 [1966].



can, as an alternative, establish the levels in the neighboring  $T_z=\pm 1$  member of the triad and search for the analog states in the  $T_z=0$  nucleus. Since there is in Be<sup>8</sup> no state that is definitely proven to

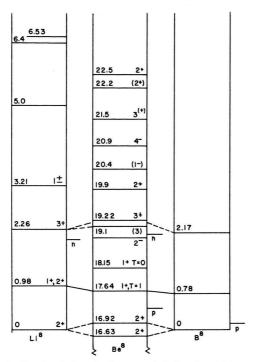


Fig. 1. Levels of the A=8 mass triad. Levels which are in doubt or do not bear on the question of isospin conservation are generally omitted. Tentative analog states are connected by a dotted line.

have T=1, we start by looking at Li<sup>8</sup>. Fig. 1 shows the level sequence and known assignments in Li<sup>8</sup> as well as some levels of interest in Be<sup>8</sup>. Table 1 lists the characteristics of states <sup>1</sup> in Li<sup>8</sup> that are established with sufficient accuracy.

The ground state of Li<sup>8</sup> has spin-parity  $2^+$  which is known unambiguously only from a complicated investigation <sup>9</sup> of the  $(\beta, \alpha)$  correlation in the beta decay to the first excited state in Be<sup>8</sup>. This spin value also results from any reasonable shell model calculation (see for instance ref. <sup>10</sup>). The assignments for the first excited state at 980 keV can be limited to the values  $1^+$ ,  $2^+$  from the  $(p, \gamma)$  correlation in the Li<sup>7</sup>(d, p)Li<sup>8\*</sup> stripping reaction <sup>11</sup>, since the measured upper limit <sup>12</sup> on the lifetime proves very pure MI character for the gamma decay. The recently discovered existence of He<sup>8</sup> and its allowed transition <sup>13</sup> to the 980-keV state support the  $1^+$  assignment.

The second excited state at 2.26 MeV is known to have  $J^{\pi}=3^{+}$  from both elastic neutron scattering <sup>14</sup> and radiative neutron capture <sup>15</sup> on Li<sup>7</sup>. The next higher state at 3.21 MeV, with a width of about 1 MeV, is observed in inelastic neutron scattering. A fit to the cross section <sup>16</sup> with reasonable reduced widths can be achieved with  $J^{\pi}=1^{\pm}$ .

The first three states mentioned above are observed in the stripping reaction <sup>17</sup> Li<sup>7</sup>(d, p). The reduced single particle widths have been computed if possible for these states from the stripping cross sections in Plane Wave Born Approximation from data at 8 MeV;

energy (Mév)	$J^{\pi}$	$\Gamma(\mathrm{keV})$	decay	$\theta_{n}^{2}$	$\Gamma_{\gamma}$
$egin{array}{c} 0 \\ 0.980 \\ 2.26 \\ 3.21 \\ 5.0 \\ 6.5 \\ \end{array}$	2+ 1+(2+) 3+ 1± (2-)	$-4 \times 10^{-14} \mathrm{sec}$ $31$ $1000$ $5$ $5$ $6$ $6$ $6$ $6$ $6$ $6$ $6$ $6$ $6$ $6$	β- γ n,γ n,n' n	0.5 0.3 * 0.09 * 0.08	$> 2.5 \times 10^{-2} \mathrm{eV}$ $0.07 \mathrm{eV}$

Table 1. Levels in Li<sup>8</sup>. \* Reduced particle widths are obtained from ref. <sup>7</sup> normalized to the DWBA value for the ground state from ref. <sup>8</sup>.

- <sup>7</sup> N. H. McFairlaine and J. B. French, Rev. Mod. Phys. 32, 567 [1960].
- <sup>8</sup> J. H. Halton and P. E. Hodgson, Proc. Rutherford Jub. Inter. Conf. (1961), p. 593.
- <sup>9</sup> K. H. LAUTERJUNG, B. SCHIMMER, U. SCHMIDT-ROHR, and H. MAIER-LEIBNITZ, Z. Phys. 155, 560 [1959].
- 10 S. Cohen and D. Kurath, Nucl. Phys. 73, 1 [1965].
- <sup>11</sup> L. F. Chase, R. G. Johnson, F. J. Vaughn, and E. K. Warburton, Phys. Rev. **127**, 859 [1962].
- <sup>12</sup> J. B. Thomas, P. Paul, S. S. Hanna, D. C. Camp, and G. A. Armantrout, Bull. Am. Phys. Soc. 11, 26 [1965].
- <sup>13</sup> A. M. Poskanzer, R. A. Esterlund, and R. McPherson, Phys. Rev. Letters 15, 1030 [1965].
- <sup>14</sup> H. B. WILLARD, J. K. BAIR, J. D. KINGTON, and H. D. COHN, Phys. Rev. **101**, 765 [1956].
- <sup>15</sup> W. L. IMHOF, R. G. JOHNSON, F. J. VAUGHN, and M. WALT, Phys. Rev. 114, 1037 [1959].
- <sup>16</sup> J. M. FREEMAN, A. M. LANE, and B. Rose, Phil. Mag. 46, 17 [1955].
- <sup>17</sup> E. W. Hamburger and J. R. Cameron, Phys. Rev. 117, 781 [1960].

the ground state width has also been analyzed with Distorted Wave Approximation 8. The values listed in Table 1 are obtained from the PWBA ratios normalized to the DWBA value for the ground state. For the 3.2-MeV state which is, because of its large width, not observed in the stripping reaction, the single particle width was computed from the scattering data for the assignments 1<sup>±</sup>. There exist no other well established levels in Li8. A possible broad level around 5 MeV with  $J^{\pi} = 2$  is only inferred from background s-wave neutron scattering 1 and possibly supported by the neutron polarization below 4 MeV. A narrow state at 6.53 MeV has been observed 1 in Li7(t, d), but its assignments remain unknown. Shell model calculations speculatively favor  $J = 4^{+}$ . This state has not been observed <sup>17</sup> in  $Li^7(d, p)$ .

Turning now to Be<sup>8</sup> to search for the analog state of the Li<sup>8</sup> ground state, one finds two candidates in the expected energy region, at 16.62 MeV and 16.92 MeV, with the spin-parity assignment  $2^+$ . While the parity of the states follows directly from their widths and related alpha break up <sup>18</sup>, the spin values have only recently been determined unambiguously from elastic alpha scattering <sup>19</sup> and less directly from the Be<sup>9</sup> (He<sup>3</sup>,  $\alpha$ ) reaction through the  $(\alpha, \alpha)$  correlation <sup>20</sup>. There are, at present, no other states known with  $J^{\pi} = 2^+$  in this energy region between the Li<sup>8</sup> and B<sup>8</sup> ground states. The question as to which of the two is the analog state with T = 1 will be discussed in the next section.

It can be seen in Fig. 1 that there are again two states found in Be<sup>8</sup> at 17.64 MeV and 18.15 MeV with  $J^{\pi} = 1^+$  that come into consideration as the analog state to the first excited state in Li<sup>8</sup>. Positive parity for the 17.6 MeV state follows from the p-wave elastic scattering of protons <sup>21</sup> on Li<sup>7</sup> at this resonance, and the p-wave stripping pattern <sup>22</sup> observed in the reaction Li<sup>7</sup>(d, n) at 5 MeV. Spin and parity are deduced from the strong ground state gamma transition in the radiative capture <sup>1</sup> of protons on Li<sup>7</sup> at 440 keV, and from the E2/M1 ratio for the

transition to the first excited state. The 17.6-MeV state is, incidentally, the only one in Be<sup>8</sup> for which some direct evidence for T=1 assignment was available until recently. This is based on the reasonably strong M1-transition to the ground state and Morpurgo's Rule <sup>23</sup>. This point will be discussed in more detail in the next section.

Positive parity for the state at 18.15 MeV follows directly from the p-wave elastic  $^{21}$  and inelastic proton scattering  $^{24}$  on  $\text{Li}^7$  around 1 MeV and again a p-wave stripping pattern  $^{22}$  in  $\text{Li}^7(\mathbf{d},\mathbf{n})$ . The presence of a strong ground state gamma transition  $^1$  and the absence of any observable alpha width make J=1 the only choice. If the spin were 2, alpha decay would be allowed (except for isotopic spin considerations). One would then expect to observe some alpha breakup, since this channel has so much phase space available.

The next analog state should have  $J=3^+$ . There are, at 19.05 MeV and 19.22 MeV two levels in the right energy region which might have correct spin and parity. A neighboring level at 18.9 MeV is assigned 2. It should be noted that spin assignments for most levels above the (p, n) threshold, i. e. above 19 MeV, are deduced from energy dependence and size of cross sections and hence not very firmly established. However, within this limitation, the pair of levels at 18.9 MeV and 19.22 MeV has been most thoroughly studied in connection with the treshold behaviour of the reaction Li<sup>7</sup>(p, n). The assignment 2 and 3 are the only ones mutually consistent 25. The level at 19.05 MeV is speculatively assigned 3<sup>+</sup>. Elastic proton scattering <sup>26</sup> gives  $J \leq 3$ . The evidence from the hard gamma decay is conflicting 27 and does not, at this time, rule out any of the above spin values. The positive parity has not been proven. It might be noted that there is some evidence 28 from  $Li^{7}(p, n)$  for another state with J=3 at 21.5 MeV. The spin assignment rests on the assumption that only one level is present in the observed cross section.

<sup>&</sup>lt;sup>18</sup> P. Paul and D. Kohler, Phys. Rev. 129, 2698 [1963].

<sup>&</sup>lt;sup>19</sup> E. Shield, H. E. Conzett, P. Darriulat, H. Pugh, and R. J. Slobodrian, Bull. Am. Phys. Soc. 9, 703 [1964].

<sup>&</sup>lt;sup>20</sup> D. J. Sullivan and P. B. Treacy, Nucl. Phys. 78, 225 [1966].

<sup>&</sup>lt;sup>21</sup> W. D. Warters, W. A. Fowler, and C. C. Lauritsen, Phys. Rev. **91**, 917 [1953].

<sup>&</sup>lt;sup>22</sup> F. S. DIETRICH and C. D. ZAFIRATOS, Bull. Am. Phys. Soc. 10, 439 [1965].

<sup>&</sup>lt;sup>23</sup> G. Morpurgo, Phys. Rev. 110, 721 [1958].

<sup>&</sup>lt;sup>24</sup> F. Mozer, W. A. Fowler, and C. C. Lauritsen, Phys. Rev. 93, 829 [1954].

<sup>&</sup>lt;sup>25</sup> H. W. Newson, R. M. Williamson, K. W. Jones, J. H. Gib-Bons, and H. Marshak, Phys. Rev. 108, 1294 [1957].

<sup>&</sup>lt;sup>26</sup> P. R. Malmberg, Phys. Rev. 101, 114 [1956].

<sup>&</sup>lt;sup>27</sup> R. R. Perry, B. Mainsbridge, and J. Rickard, Nucl. Phys. 45, 586 [1963]; V. Riech, Phys. Letters 6, 267 [1963].

<sup>&</sup>lt;sup>28</sup> R. R. Borchers and C. H. Poppe, Phys. Rev. **129**, 2679 [1962].

The higher levels in  ${\rm Li^8}$  are not known well enough to allow a detailed search for analog states. For both  $J=1^\pm$  and  $J=2^-$  assignments levels can be found in Be<sup>8</sup>. But only the levels with  $2^-$  at 18.9 MeV and with  $1^-$  at 20.35 MeV are reasonably certain. The first has been assigned from the cross section dependence of the  ${\rm Li^7}(p,n)$  threshold  $^{25}$ , the latter from the  ${\rm Li^7}(p,n')$  threshold  $^{29}$ . All other states except the ones decaying into the alpha channel lack definite values for any spin and parity.

## II. Isopin of the 2+ and 1+ States

Both in Li<sup>8</sup> and Be<sup>8</sup> the states with  $J^{\pi}=2^{+}$  and  $1^{+}$  have ben studied extensively and their properties are sufficiently well known to allow a detailed comparison of analog states. In Be<sup>8</sup> there are two states to choose from for each Li<sup>8</sup> state. The following is a discussion of the experimental data available bearing on the question of isospin character of these states.

Isospin conservation demands that a state with T=1 in Be<sup>8</sup> cannot break up into two alpha particles. Since this is the only channel energetically open for the  $2^+$  states and since if allowed, it has 17 MeV available for the breakup, one expects the width of

the T=0 state to be much larger than that of the state with T=1. In fact, however, Table 2 shows that the experimental widths are both very narrow and about equal to 90 keV. Of course, this alone does not say much about the isospin character of these states since there might well be a small ( $\sim 10\%$ ) T=0 admixture in the T=1 state, and possibly a particular configuration in the T=0 state inhibiting alpha decay. In the case of the  $1^+$  states alpha decay is forbidden by additional symmetry requirements and their widths are due only to single nucleon channels. But these are impartial to isospin. As a check for the analog states, therefore, a number of experiments have been performed which test more or less directly the isospin character of the four states:

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#### a) Particle reactions

Much work has been done investigating the  $2^+$  and  $1^+$  states in particle reactions. The observed widths of the states and the respective feeding cross sections (sometimes relative), are listed in Table 2. Below we treat in more detail the reactions that are specifically sensitive to the isospin.

The oldest information (aside from the capture reactions) about the isospin of these four states comes

(Energy MeV)		$16.6\ (2^+)$		16.9 (2+)		17.6 (1+)		18.1 (1+)	
Ref.	Reaction	$\Gamma({ m keV})$	σ	$\Gamma({ m keV})$	σ	$\Gamma({ m keV})$	σ	$\Gamma({ m keV})$	σ
39	$\mathrm{C}^{12}(\gamma\alpha)$ 20 MeV	< 300	~ ½*	< 300	1*	< 300	~ ½*		0*
30	$B^{10}(d\alpha)$ 8 MeV	$95 \pm 20$	11.3 mb		$9.8 \mathrm{mb}$		0.7 mb		8.3 mb
31	$21~{ m MeV}$		1 *		1 *		_		
22	$\mathrm{Li}^{7}(\mathrm{dn})$		$23~\mu \mathrm{b/sr}$		$1 \mu b/sr$		$7 \mu b/sr$		$10 \mu b/sr$
32	$\mathrm{Li}^6(\mathrm{Li}^6lpha)$		1 *		1 *		0 *		~ ½ *
21	Li <sup>7</sup> (pp)					12		168	
1, 33, 34	$\mathrm{Li}^{7}(\mathrm{p}\gamma)$	$85\pm15$	$6 \mu b^+$	$90 \pm 15$	$0.4~\mu b^+$	12	_	168	_
35	$Be^{9}(He^{3}\alpha)$	$96 \pm 20$	,	80 + 15					
36	3.8 MeV	105 + 30	1 *	88 + 25	10*	< 15	5*		
20	3 MeV	1	1 *		2*	F	_		_
36	$Li^6(He^3p)$	$95\pm20$	3 *	85 + 20	3*	< 20	1*		_
1	( 17)	$82 \pm 6$		$93 \stackrel{-}{\pm} 7$			_		_
37	$Be^{9}(pd)$		1*		> 25 *		~ 3.5 **		1**
38	$Be^{9}(dT)$		weak		strong		strong		0

Table 2. Widths and cross sections for the states with  $J^{\pi}=2^+$  and  $1^+$ . Numbers marked with \* or \*\* are relative to each other. in each row. \* These cross sections refer to the transition from the 17.6-MeV state.

<sup>&</sup>lt;sup>29</sup> S. G. Buccino, C. E. Hollandsworth, and P. R. Bevington, Nucl. Phys. 53, 375 [1964].

<sup>30</sup> C. P. Browne and J. R. Erskine, Phys. Rev. 143, 683 [1966].

<sup>&</sup>lt;sup>31</sup> P. D. PARKER and P. F. DONOVAN, Bull. Am. Phys. Soc. 10, 1135 [1965].

<sup>&</sup>lt;sup>32</sup> R. R. CARLSON and K. G. KIBLER, University of Iowa Report SUI-64-21, (1964).

<sup>&</sup>lt;sup>33</sup> P. PAUL, S. L. BLATT, and D. KOHLER, Phys. Letters **10**, 201 [1964]; D. KOHLER and P. PAUL, Phys. Letters **15**, 157 [1965].

<sup>&</sup>lt;sup>34</sup> M. Wilson and J. B. Marion, Phys. Letters **14**, 313 [1965]; J. B. Marion and M. Wilson, Nucl. Phys. **77**, 129 [1966].

<sup>35</sup> W. E. Dornbusch and C. P. Browne, Phys. Rev. 131, 1212 [1963].

<sup>&</sup>lt;sup>36</sup> J. R. Erskine and C. P. Browne, Phys. Rev. **123**, 958 [1961].

<sup>&</sup>lt;sup>37</sup> J. B. Marion and Carl A. Ludemann, Bull. Am. Phys. Soc. 11, 26 [1966].

<sup>&</sup>lt;sup>38</sup> G. T. GARVEY, J. CERNY, and H. Pugh, Bull. Am. Phys. Soc. 11, 26 [1965].

from the work of Govard and Wilkins 39 studying final states in the reaction  $C^{12}(\gamma, \alpha) Be^8 \rightarrow 2 \alpha$  at y-energies around 20 MeV. They observe a peak at 16.9 MeV with an indication of a 30% contribution at 16.6 MeV. The transition to the 17.6-MeV state is clearly present. The angular distributions indicate El  $\gamma$ -absorption and  $J^{\pi} = 2^+$  for the final states at 16.9 MeV and 16.6 MeV. However, in a self-conjugate nucleus such as C12 the matrix element for E1 gamma transitions vanishes in the one particle transition approximation 23 unless  $\Delta T = \pm 1$ . Therefore, El gamma absorption leads to compound states with T=1. The subsequent  $\alpha$ -decay preserves the isospin character. It follows that the states in Be8 observed in this reaction must have at least a strong T=1component. The fact that the 18.15-MeV state is not observed indicates T=0.

Browne and Erskine 30, 36 investigated the reaction B<sup>10</sup>(d, a)Be<sup>8</sup> at 4 and 8 MeV. Since all particles involved have T=0, this reaction should populate only final states with T=0 unless the deuteron energy happens to be close to a compound state with T=1. Therefore, Parker and Donovan <sup>31</sup> repeated this experiment at 20 MeV where the interaction should be entirely direct and COULOMB mixing in the intermediate system unimportant. The results at all bombarding energies are qualitatively the same, namely, that both states at 16.6 and 16.9 MeV are populated with about equal strength. The actual cross sections for the four final states, obtained at 8 MeV, are listed in Table 2. The angular distributions 30 to both 2+ states are very similar, indicating similar wave functions. The fact that both states are populated with equal and strong cross sections points to large T=0 components. The state at 17.6 MeV for which there exists independent evidence for a good T=1 assignment (see Sect. c) is populated much less strongly than the "partner" at 18.15 MeV. Again angular distributions are not dissimilar. In comparing the cross sections for the 1 and the 2 states it should be noted that the cross sections for the latter ones are enhanced by a statistical factor 5/3 over the 1<sup>+</sup> states. The reduced transition strength to the 2<sup>+</sup> states is then about 1/2 of that to the 18.15 MeV state. This experiment indicates that the 2+ states must both have a large, about 50%, T=0 admixture while the 1 states are divided into one with predominantly T=1 at 17.6 MeV and one with predominantly T=0 at 18 MeV.

Carlson and Kibler <sup>32</sup> studied the reaction Li<sup>6</sup> (Li<sup>6</sup>,  $\alpha$ )Be<sup>8</sup> at 3 MeV. This reaction again involves only particles with zero isospin. Both states at 16.6 and 16.9 MeV are populated with about equal strength. The state at 18.15 MeV is observed with somewhat less intensity and the 17.6 MeV state not at all within the errors. This again suggests strong isospin mixing in the 2<sup>+</sup> pair and essentially isospin conservation in the 1<sup>+</sup> states. However, these results may not be quite conclusive in themselves in view of the low Li<sup>6</sup> bombarding energy and larger charge which might induce Coulomb effects during the reaction <sup>32</sup>.

The reactions Li<sup>7</sup>(d, n)Be<sup>8</sup> and Li<sup>7</sup>(d, p)Li<sup>8</sup> are direct analog reactions and the relative cross sections, therefore, can be used to compare the ground state and first excited state of Li<sup>8</sup> on one side, and the states in Be<sup>8</sup> on the other side. The Li<sup>7</sup>(d, n) reaction cross section <sup>22</sup> shows a good stripping pattern for the 16.6-, 17.6- and 18.15-MeV states and only a small essentially isotropic cross section for the 16.9-MeV state. The reduced stripping widths for the three stripping states <sup>40</sup> are given in Table 3. The

$(\mathrm{MeV})$	$J^{\pi}$	$\Gamma \  m (keV)$	$\Gamma_{ m p.} \  m (keV)$	$ heta_{ m p}^2$	$\Gamma_{\gamma}(0)$ (eV)	$\Gamma_{ m  u}(2.9) \  m (eV)$	breakup channel
16.62	2+	85	_	2.5 *	_	_	α
16.92	2+	90	_	0*	_	_	α
17.64	1+	12	-	0.13 1*	16.7	8.2	n,p
18.15	1+	168	6	0.25 1.90*	2.7	2.7	n,p, n', p'

Table 3. Some properties of the 2<sup>+</sup> and 1<sup>+</sup> states around 17 MeV in Be<sup>8</sup>. Reduced widths \* are from ref. <sup>40</sup> and relative to each other.

extraction of stripping reduced widths for the single particle unbound states at 17.6 and 18.15 MeV is problematic. But for these states reduced particle widths can also be obtained from the total widths and the partial proton widths measured in the scattering and capture of protons on Li<sup>7</sup>. The ratios of the latter values agree well with the ratios of the stripping reduced widths. Although the absolute values of reduced particle widths extracted from stripping cross sections are believed to be correct

<sup>&</sup>lt;sup>39</sup> F. K. Goward and J. J. Wilkins, Proc. Roy. Soc. London 228, 376 [1955].

<sup>&</sup>lt;sup>40</sup> F. Dietrich, private communication, 1965 (these are PWBA values, insofar preliminary).

only within a factor of 2, ratios of reduced widths between different levels might be considered more accurate. The ratio  $^{40}$   $\Theta_{\rm p}^2(16.6)/\Theta_{\rm p}^2(17.6)=2.50$ ,  $\Theta^2(16.9)\approx 0$  can be compared to the ratio of  $\Theta_{\rm n}^2({\rm ground})/\Theta_{\rm n}^2(.980)=1.89$  in Li<sup>8</sup> from the reaction Li<sup>7</sup>(d, p). For maximal isospin mixing one expects  $\Theta_{\rm p}^2(16.6)/\Theta_{\rm p}^2(17.6)\cong 3.8$ . The observed ratio  $^{40}$  does not decide between maximally mixed or pure isospin but favors T=1 for the 16.6-MeV state and T=0 for the one at 16.9 MeV. Theoretically, one expects a ratio of  $\Theta_{\rm p}^2(16.9)/\Theta_{\rm p}^2(16.6)=3$  for states with T=0 at 16.9 MeV and T=1 at 16.6 MeV. The experimental value of  $\Theta_{\rm p}^2\approx 0$  for the 16.9-MeV state can be explained as a structure effect which will be discussed in the following section.

Another experiment which directly compares the Li<sup>8</sup> states with the ones in Be<sup>8</sup> was performed by Garvey et al. 38. They studied simultaneously the reactions Be<sup>9</sup>(d, t)Be<sup>8</sup> and Be<sup>9</sup>(d, He<sup>3</sup>)Li<sup>8</sup>. As in the previous case the relative strength of analog final states in Be8 and Li8 is given by a simple ratio of isospin coupling coefficients if isospin is a good quantum number. For the present case one expects  $\Theta_n^2(\text{Li}^8)/\Theta_n^2(\text{Be}^8)=2$ , by coupling a proton to Li<sup>8</sup> and a neutron to Be8 in the inverse reaction. The experiment 38 yields a value of 2.2 which is in good agreement with isospin conservation for the 980-keV state in Li<sup>8</sup> and the 17.6-MeV state in Be<sup>8</sup>. The ratio for the Li8-ground state and the 16.9-MeV state is larger, the one to the 16.6-MeV state much smaller than expected. Hence neither state can be strictly the analog state of the Li<sup>8</sup> ground state. In section III, it will be shown that the preference for the state at 16.9 MeV can again be explained by its structure.

#### b) Beta decay

The  $\beta$ -decay from B<sup>8</sup> has a Q value high enough to allow population of both the 16.6- and 16.9-MeV state. The transition to the analog of the B<sup>8</sup> ground state is a superallowed transition. Its matrix element should therefore, in general, be larger than the one for the transition to the T=0 state. For a superallowed transition  $2^+ \rightarrow 2^+$  one expects an ft value of about 1000. This calculation, however, depends on the wave function of initial and final state through

the allowed axial vector part of the transition. The β-decay was measured by Pfander et al. 41 who detected the delayed alpha particles from the breakup of the final states. Their data are shown in Fig. 2. The peak due to a transition to the state at 16.6 MeV is clearly resolved. From the total alpha spectrum they obtain an ft value of  $\log t = 2.9$  which is in very good agreement with the theoretical expectation for a super-allowed transition. It should be noted that this result is very crucial because it offers the most direct check on the presence of a large T=1component in either one or both of the two 2<sup>+</sup> states. It appears, however, that with the proper wave functions in the present case the isospin allowed matrix element is not theoretically expected 42 to be substantially larger (~ factor 2) than that for the  $\Delta T = 1$  transition. Under these circumstances the effect of maximal isospin mixing in the 2+ states would change the ft value for the transition to either level by only about 30%. This requires then a very precise determination of the ft value. Fig. 2 shows that the analysis presented in ref. 41, assuming only the presence of the 16.6-MeV state, requires a width for this state of 150 keV which is much larger than observed in any other reaction (see Table 2). If the correct width of about 90 keV is used only about half of the intensity in the lower part of the spectrum is accounted for. However, if there is any transition to the state at 16.9 MeV, the very steep phase space dependence of  $\beta$ -decay would suppress most of it above and even down in the peak region of the 16.6-MeV state. The data of ref. 41 have been reanalyzed assuming equally strong transition matrix elements to both states and about equal widths. The same procedure given in ref. 41 was used. As Fig. 2 shows, these assumptions yield good agreement with the data. Of course, the fit is most likely not unique but within small limits the width of one state can be exchanged for the amplitude of the other. The ft value which was obtained from the total spectrum is now shared by both transitions giving about half the superallowed strength to each of them. This result is interpreted to the effect that the beta-decay does not rule out, but indeed favors the hypothesis that both  $2^+$  states have components with T=1 in their wave functions. More precise data in the region above the 16.6-MeV state are clearly desirable.

<sup>&</sup>lt;sup>41</sup> E. Matt, H. Pfander, H. Rieseberg, and V. Soegel, Phys. Letters 9, 174 [1964].

<sup>&</sup>lt;sup>42</sup> J. B. Marion, private communication.

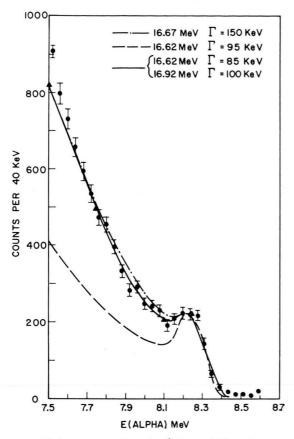


Fig. 2. Alpha spectrum from the  $\beta$ -decay of  $B^8$  to the states at 16.6 MeV and 16.9 MeV in  $Be^8$ , taken from the work of Matt et al. <sup>41</sup>. Various fits to the data are indicated. A fit with the assumption of equal matrix elements to the 16.6-MeV and 16.9-MeV states appears to yield good agreement (see text).

## c) Electromagnetic transitions

Another check on the isospin character of the 2<sup>+</sup> and 1<sup>+</sup> states in Be<sup>8</sup> again makes use of the fact that in a self-conjugate nucleus electromagnetic dipole

transitions follow an isospin selection rule. In the present case an approximate rule 23 says that M1transitions with  $\Delta T = 1$  should be enhanced by at least a factor 50. This rule has been shown to be obeyed very well in the entire p-shell. In the case of the 1<sup>+</sup> states the rule applies to the transitions to the ground state and the M1-components of the transitions to the 2<sup>+</sup> states. Some observed radiative transition strengths are listed in Table 3. On the basis of this rule the strong ground state transition 1 from the 17.6-MeV state is quite consistent with the assignment T=1, in particular since the large <sup>33</sup>P[31] component in the wave function of the 1+ state does not participate in the transition to the <sup>11</sup>S[4] ground state. The same agreement holds for the M1 part of the transition to the 2<sup>+</sup> first excited state <sup>1</sup>. The ground state transition from the 18.15-MeV state is too strong for a pure T = 0 assignment and does not rule out T = 1. It may, however, result from a small T=1 isospin admixture in the state.

The gamma transitions from the 2<sup>+</sup> states have not been observed due to the width of the states and the fact that the capture reaction  $He^4(\alpha, \gamma)$  poses experimental problems. However the gamma transitions to the 2+ states from the 1+ states have been measured by Paul et al. 33, 43, and Wilson and Marion <sup>34</sup> in the capture reaction Li<sup>7</sup>(p,  $\gamma$ ). The latter group detected the alpha particles from the final state break up; the first group measured the gamma transition in coincidence with the alpha particles. The excitation function of the transitions to both final states is shown in Fig. 3. The electromagnetic widths for both transitions from the two 1 states are listed in Table 4. The strong transition  $17.6 \rightarrow 16.6$  of  $2 \times 10^{-2}$  eV or 1 Weisskopf unit indicates predominant M1 character and hence a

Transition	$\Gamma_{\gamma}(10^{-2}{\rm eV}) \ 17.6  ightarrow 16.6$	Ratio $17.6 \to 16.9$ $17.6 \to 16.6$	$\Gamma_{\gamma}(10^{-2}\mathrm{eV})$ $18.15 \to 16.6$	Ratio $18.15 \rightarrow 16.9$ $18.15 \rightarrow 16.6$
$3/2 \rightarrow 3/2$ $1/2 \rightarrow 3/2$ $1/2 \rightarrow 1/2$ $1/2 \rightarrow 1/2$ exp. $W (M1)$ $W (E2)$	$\begin{array}{c} 7.0 & (1.85) \\ 2.06 & (1.4) \\ 0.06 & (0.36) \\ 2.1 & \pm & 0.2 \\ 2.2 \\ 10^{-4} \end{array}$	$0.074$ $0.25$ $2.00$ $0.07 \pm 0.02$ $0.342$	$11.8 \\ 6.95 \\ 0.13 \\ 5.0 \\ 7.4 \\ 2 \times 10^{-3}$	$egin{array}{c} 0.16 \\ 0.43 \\ 3.1 \\ 0.45 \pm 0.1 \\ 0.64 \\ - \end{array}$

Table 4. Gamma transitions from the  $1^+$  states to the  $2^+$  states. The first three rows give the transition strengths for the indicated single particle transitions, for protons and neutrons (in parenthesis). The ratios are computed for a neutron transition divided by a proton transition. W(M1) and W(E2) are the Weisskopf widths for M1 and E2 transitions.

<sup>43</sup> P. PAUL, D. KOHLER, and K. A. SNOVER, Bull. Am. Phys. Soc. 11, 26 [1966].

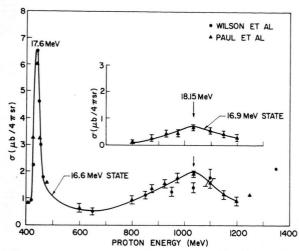


Fig. 3. Excitation function of the reaction Li<sup>7</sup>(p, $\gamma$ )Be<sup>8</sup> leading to the states at 16.6 MeV and 16.9 MeV. The data by Wilson et al. <sup>34</sup> are obtained by detecting the alpha particles from final state breakup. The curve by Paul et al. <sup>33, 43</sup> results from observation of the gammas in coincidence with the final state alpha particles; the cross section is normalized to the Li<sup>7</sup>(pa)a cross section of ref. <sup>34</sup>. At the 440-keV resonance the ratio  $(17.6 \rightarrow 16.9)/(17.6 \rightarrow 16.6) = 0.07 \pm 0.02$ .

strong T = 0 component in the final state, based on the assumption that the 17.6-MeV state has rather pure isospin T=1. In fact, it was this reaction in conjunction with the beta decay data that led to the conjecture of large isospin mixing in the 16.6-MeV state. The rather pure M1 character of the transition, already strongly in evidence from its speed 33, has been verified by Sweeney and Marion 44 by a measurement of the  $(p \gamma \alpha)$  correlation yielding E2/M1 = 0.5% substantiating applicability of Mor-PURGo's Rule. Since there is only a 7% branch from the 17.6-MeV resonance to the 16.9-MeV state, the latter could conceivably have isospin T = 1. On the other hand the equally strong transition  $18.15 \rightarrow 16.6$ indicates a large T=1 component in the 16.6-MeV state. This assumes that the 18.15-MeV level has rather pure T = 0. Again, the transition  $18.15 \rightarrow 16.9$ which is now 45% of the 16.6-MeV transition could be interpreted as evidence for a T=1 character of the 16.9-MeV state. The combined evidence from all observed γ-transitions therefore indicates strong isospin mixing in at least the 16.6-MeV state, while the 16.9-MeV state could have rather pure T=1character.

Summarizing the results of all isospin selective reactions, one is led to the conclusion that neither

of the two states at  $16.6\,\mathrm{MeV}$  and  $16.9\,\mathrm{MeV}$  can have a pure isospin character. Rather, both must contain T=1 and T=0 in amounts exceeding 30% (amplitude square).

At this point one might ask whether there could not exist another, hitherto unobserved, state with very small width, situated on the side of one of the other 2<sup>+</sup> states, which is the true analog state to the Li<sup>8</sup> ground state. This would require three states with  $J = 2^+$  within 400 keV, which would indeed be an unlikely accident. The shell model does not provide such a state. Beside this point, however, there are also the experimental arguments that such an additional state has not been observed in the many different particle reactions that have so far been studied. These include, beside the ones mentioned above, reactions that populate both isospin states equally such as <sup>23</sup> Be<sup>9</sup> (He<sup>3</sup>,  $\alpha$ ) and Li<sup>6</sup> (He<sup>3</sup>, p). Finally, it should be noted that the  $C^{12}(\gamma, \alpha)$ experiment 39 as well as the studies of the beta and gamma transitions definitely establish large T=1components in both 2<sup>+</sup> states. Thus, a hypothetical state must mix extensively and therefore have a width between 10 and 100 keV. There is no reason, barring future directly contradictory experimental evidence, not to assume that the T=1 strength of the Li<sup>8</sup> ground state is contained in the states at 16.6 and 16.9 MeV.

## III. Single Particle Configurations and Isospin Mixing

The following makes use of cluster model to describe the essential features of some states in Be<sup>8</sup>. This is done with full knowledge of the limitations of such model with the intent to establish easily useful relationships between experimental data and theoretical character of the states involved. The implications are, of course, that such cluster configurations will indeed in a more rigorous shell model calculation turn out naturally to be the predominant parentage.

In view of the experimental evidence that both  $2^+$  states have large T=0 components which allows break-up into two alpha particles, one wonders why both states are in fact so narrow. The width of 90 keV corresponds to a reduced  $\alpha$ -width of 1.5% of the Wigner limit. On the basis of the strong gamma transition to the 16.6-MeV state it was first proposed by Paul et al.  $^{33}$  that this state have a configuration

<sup>44</sup> W. E. Sweeney Jr. and J. B. Marion, Phys. Letters 19, 243 [1965].

(Li<sup>7</sup>+p) which could explain the small overlap with two alphas. At the same time it accounts for the large overlap with the 17.6-MeV state indicated by the M1 transition. The latter state is known from both the proton width obtained in the (Li+p) reactions and the reduced Li<sup>7</sup>(d, n) stripping width to have  $\Theta_p^2 = 0.130$ , which corresponds to 1/4 of an average single particle width <sup>45</sup> in the p-shell <sup>46</sup>. The stripping reduced width extracted from the cross section <sup>40</sup> of the reaction Li<sup>7</sup>(d, n)Be<sup>8</sup> is a direct measure of the amount of (Li<sup>7</sup>+p) configuration present. The measured ratio is  $\Theta_p^2(16.63)/\Theta_p^2(17.64) = 2.50$  (for equal stripping radii and PWBA) which indicates indeed about a single particle value for the 16.6-MeV state.

There is another way to put directly a proton into the final state, i. e. direct radiative capture of a proton by Li<sup>7</sup>. In this transition from the continuum state, in lowest order, an s-wave proton radiates its energy off directly and is captured into the final state. If the proton loses all its energy essentially in the extranuclear Coulomb field and is captured into a bound final state, the cross section can be calculated easily in the approximations of Christy and Duck 47. The present case is, because of its low proton and gamma energy (neglecting the influence of the small alpha width) ideally suited for a calculation of this process 43 in the above approximation. One obtains a good fit for the nonresonant background in the reaction  $Li^7(p, \gamma)Be^8(16.6)$  $E_{\rm p} = 800 \text{ keV for a reduced proton width } \Theta_{\rm p}^2(16.6)$ =0.3-0.5. This is in excellent agreement with the stripping reduced width. The single particle character, more specifically the configuration  $(Li^7 + p)$  (where the proton may of course be  $p_{1/2}$  or  $p_{3/2}$ ) is therefore well founded for the 16.6-MeV state. The predominance of this configuration does indeed make sense since these states, to be able to contain a T=1 component, have to break up an alpha particle and are energetically close to the single particle Q value but still several MeV below more complicated channels such as  $(He^5 + He^3)$ , etc.

Marion realized first  $^{48}$  that this picture could be extended to explain the large isospin mixing in the  $2^+$  states by assuming a configuration (Be<sup>7</sup> + n) for the state at 16.9 MeV. He showed that with these

$$\chi_0 = \frac{1}{\sqrt{2}} (\chi_\beta - \chi_\alpha)$$
$$\chi_1 = \frac{1}{\sqrt{2}} (\chi_\beta + \chi_\alpha)$$

having eigenvalues T=0 and T=1, respectively. If we next form new eigenstates maximally mixed in isospin

$$\chi_{+} = \frac{1}{V^2} (\chi_0 + \chi_1) = \chi_\beta$$

$$\chi_{-} = \frac{1}{V^2} (\chi_0 - \chi_1) = \chi_\alpha$$

we obtain just the separation into the two single particle configurations. Of course, these considerations are not limited to single particle configurations but apply similarly to other clusters. If this description is correct and the 16.6-MeV state is maximally mixed in isospin with a single particle configuration  $(Li^7 + p)$ , then the 16.9-MeV state should have a single particle neutron width belonging to the configuration  $(Be^7 + n)$ . This is supported by the Li7(d, n) reaction 22 which shows a good stripping pattern for the 16.6-MeV state but a very small and isotropic cross section for the 16.9-MeV state. A direct experimental proof is offered by a study of the analog reaction Be7(d, p)Be8 which should populate the 16.9-MeV state predominantly. However, the difficulties involved with a Be7 target have so far not made this check feasible.

There is no other direct way of adding a neutron on to Be<sup>7</sup>. But there are several indirect ways by which the neutron reduced widths of the 16.6-MeV and 16.9-MeV states can be checked. They all involve pickup of a neutron out of a Be<sup>9</sup> nucleus. It can be seen very easily that from the predominant cluster configuration of the Be<sup>9</sup> ground state, i. e.  $\alpha + \alpha + n$ , one cannot reach the T = 1 eigenstates in Be<sup>8</sup> by

two configurations most particle reaction data for the  $2^+$  states can be explained. Remaining within the single particle picture, it is physically obvious that a state with definite isospin has to contain the configurations  $\psi_{\alpha} = \text{Li}^7 + p$  and  $\psi_{\beta} = \text{Be}^7 + n$  in equal amounts. Any isospin mixing shifts the balance toward  $\alpha$  or  $\beta$ . In a more formal way, we start with the two configurations  $\chi_{\alpha}(\text{Li}^7 + p)$  and  $\chi_{\beta}(\text{Be}^7 + n)$  to form states of definite isospin

<sup>45</sup> We always refer to the average single particle reduced width in the p-shell which amounts to about 0.5-0.6 of the Wigner limit.

<sup>&</sup>lt;sup>46</sup> A. M. Lane, Rev. Mod. Phys. 32, 519 [1960].

 <sup>&</sup>lt;sup>47</sup> R. F. Christy and I. Duck, Nucl. Phys. 24, 89 [1961].
 <sup>48</sup> J. B. Marion, Phys. Letters 14, 315 [1965].

pickup of the outer neutron. However, if the two alphas are symbolically replaced by the possible cluster configurations  $(Be^7+n+n)\pm(Li^7+p+n)$  one can by pickup of either an inner neutron or proton form  $Be^7+n$  and  $Li^7+n$ . This is precisely the experiment of Garvey et al. <sup>38</sup> in which  $Be^9(d,t)Be^8$  and  $Be^9(d,He^3)Li^8$  are compared. The (d,t) reaction picks up a neutron and the predominant population of the 16.9-MeV state over the 16.6-MeV state is, in this picture, explained by the large  $(Be^7+n)$  component in the upper state. This supports very nicely Marion's model.

Marion and Ludemann 37 studied the reaction Be<sup>9</sup>(p, d) at 40 MeV which involves the same pickup process. The same holds for the reaction  $Be^9$  (He<sup>3</sup>,  $\alpha$ ) in which case, however, an a-particle knock out process is possible 48 in addition to the neuron pickup. It can easily be seen that by knocking out one of the initial alphas the remaining alpha and the projectile He<sup>8</sup> can combine with the outer neutron again to Be<sup>7</sup> + n. As can be seen in Table 2 both reactions favor heavily the 16.9-MeV level over the one at 16.6 MeV, in the first case by about 25:1, in the second 36 by 10:1. Summing up the evidence, all particle reactions support Marion's model for the 2<sup>+</sup> states. We reserve the discussion of the electromagnetic transitions in this context for the next section.

We have, so far, always assumed maximal splitting. The total width of these states is due to alpha decay only, which in turn depends in both states on the T=0 component. It is assumed that both states have isospin mixed but otherwise similar wavefunctions. It is then easy to verify that the ratio of the widths is directly equal to the ratio of the squares of the T = 0 isospin amplitudes in both states. As can be seen from Table 2 there is at present, within the accuracy of the measured widths, no reason to depart from maximal mixing. The mixing also reflects the relative ratios of reduced neutron and proton widths in these states. However, in view of the uncertainties encountered in obtaining reliable reduced widths even from deuteron stripping reactions it seems that for a comparison of different states one might overstress the cluster picture by attempting more than the semiquantitative arguments given above.

One can also test the single particle picture for the 1<sup>+</sup> states. Since all evidence points to rather good (within 10%) isospin purity in these states they should have about equal neutron and proton reduced

widths, each with half a single particle strength. The stripping reduced width ratio  $\Theta_{\rm p}^2(17.6)/\Theta_{\rm p}^2(16.6)$ = 0.4 obtained 40 from Li<sup>7</sup>(d, n) is in nice agreement with this model. Since both 1<sup>+</sup> states are neutron stable, the neutron reduced width can, again, only be checked by a pickup reaction. The reaction Be<sup>9</sup> (He<sup>3</sup>, α)Be<sup>8</sup> populates the 17.6 MeV-state about half as strongly as the 16.9 MeV state as predicted by the model (see Table 2). Thus, the extreme single particle picture seems to hold for the 1<sup>+</sup> doublet as well as for the 2<sup>+</sup> states. A surprising fact is, however, that the proton width of the 18.15-MeV state is almost twice the width of the 17.6-MeV state. We attribute this to a larger radius for the 18.15-MeV state due to the fact that the proton is less tightly bound because the state lies higher above the  $(Li^7 + p)$ threshold.

Finally, the question is whether such a single particle model is also appropriate for the low states in Li<sup>8</sup>. Since reduced widths listed in Table 1 are of single particle size, this is indeed so. Indirectly, this is also proven by the observed ratios <sup>38</sup> of the reaction cross sections of Be<sup>9</sup>(d, He<sup>3</sup>)Li<sup>8</sup> and Be<sup>9</sup>(d,t)Be<sup>8</sup>. The experimental value of 2 for the ratio of the 980-keV state in Li<sup>8</sup> and the 17.6-MeV state in Be<sup>8</sup> indicates similar wave functions for both states. This result is, of course, contained in the statement that these two states are true isospin analog states.

## IV. Models for Be8

The last section has shown that for the states at 16.6 MeV and 16.9 MeV Marion's hypothesis of maximally isospin-mixed single particle states is a very successful one. We will, therefore, in the following assume such configurations (Li<sup>7</sup>+p) for the 16.6-MeV and  $(Be^7 + n)$  for the 16.9-MeV state, where the single particle may be  $p_{1/2}$  or  $p_{3/2}$ . There are then, at least two ways to extend the model to higher levels. One way assumes total breakdown of isospin in Be8. On this basis, Marion 49 had, at an early stage, proposed core excited configurations  $(Li^{7*}+p)$  and  $(Be^{7*}+n)$  for the 17.6-MeV and 18.15-MeV state, respectively. Such a model splits the A=8 triad into two T=1/2 doublets. On one side in Be<sup>8</sup> the states with configurations (Li<sup>7</sup>+p),  $(Li^{7*}+p)$ ,  $(Li^{7**}+p)$  etc. and their analog states in Li<sup>8</sup> with  $(Li^7 + n)$ ,  $(Li^{7*} + n)$ ... on the other side in Be<sup>8</sup> the states with  $(Be^7 + n)$ ,  $(Be^{7*} + n)$ ... and the

<sup>&</sup>lt;sup>49</sup> J. B. Marion, Bull. Am. Phys. Soc. 10, 9 [1965].

analog states in  $B^8$  with  $(Be^7+p)$ ,  $(Be^{7*}+p)$  etc. Such a model has certain attractive features: The states in  $Li^7$  provide the correct spin sequence and level spacing <sup>1</sup> (within about 1 MeV) up to the  $3^+$  states. The lifetime of the first excited state <sup>46</sup> in  $Li^7$ , scaled up by the correct phase space factor gives within a factor of 2 the correct strength for the transition  $17.6 \rightarrow 16.6$   $(5.2 \times 10^{-2} \text{ eV})$  vs. experimental  $2.2 \times 10^{-2} \text{ eV})$  while the weak transition  $17.6 \rightarrow 16.9$  in this model is explained by the very bad overlap of the initial and final state wave functions.

However, as outlined in the previous sections, there is evidence against large isospin mixing in the 1 states. But the model can be tested even more specifically through the gamma transitions from the 18.15-MeV state to the 2<sup>+</sup> doublet. This topic is dealt with in detail in a forthcoming publication 50 and we give here only the main arguments. If the extreme core excitation model (i. e. the outer particle treated as spectator only) is correct, the  $18.15 \rightarrow 16.9$  and  $18.15 \rightarrow 16.6$  decays should be mirror transitions to the  $17.6 \rightarrow 16.6$  and  $17.6 \rightarrow 16.9$  decays. The experimental gamma widths listed in Table 4 do not support this since they give the  $18.15 \rightarrow 16.6$  as the stronger transition from the upper state. (It should, however, be noted that there exists at this time still some degree of disagreement 34 about the decay from the 18.15-MeV level). The large proton reduced width of the 18.15-MeV state is, of course, the strongest argument against such a pure neutron configuration. To conserve isospin for the 1<sup>+</sup> states one is compelled to introduce the charge conjugated configuration into each state, i. e.  $(Li^{7*} + p) \pm (Be^{7*} + n)$ . Even with these wave functions it does not suffice to assign to the single particle a spectator role only, because then the relative transition strengths of  $17.6 \rightarrow 16.6$  and  $17.6 \rightarrow 16.9$  should be roughly equal to the inverse ratio of the Li7\* and Be7\* lifetimes <sup>51</sup>, i. e. 1.85. The experimental value is 15. Conceivably, coherent decay of the core and the valence particle could explain the observed transition strengths.

The good stripping pattern observed in Li<sup>7</sup>(d, n) for the 1<sup>+</sup> states and the large stripping widths extracted from the cross sections constitute some evidence against a predominantly core excited con-

figuration. These would require a two-step process. In the case of the 18.15-MeV state the p and p' channels are open and the ratio of the particle widths  $\Gamma_{\rm p}/\Gamma_{\rm p'}$  can be measured as a direct check on the amount of  $({\rm Li}^{7*}+{\rm p})$  configurations present. Experimentally  $\Gamma_{\rm p}/\Gamma_{\rm p'}=27$  and this, assuming p-wave only, leads to  $\Theta_{\rm p}{}^2/\Theta_{\rm p'}{}^2\cong 5$ . Obviously then it does not suffice to consider core excited clusters only but one needs also strong  ${\rm Li}^7$  ground state clusters in the  ${\rm I}^+$  states. Therefore, the experimental data cannot be explained by a simple cluster picture built on one primary configuration.

An alternate explanation for the large isospin mixing in the  $2^+$  states was first proposed by Kohler and Paul <sup>33</sup>, and put on a more theoretical basis by Barker <sup>52</sup>. In this model, the basis for the large isospin mixing is the accidental initial energy degeneracy of two states with  $J^{\pi}=2^+$ . Before Coulomb forces are turned on these states have similar initial wave functions  $\chi_0$  and  $\chi_1$  with isospin T=0 and T=1. The Coulomb interaction  $V_c$  removes the degeneracy and mixes the two wave functions maximally as described in the previous section, with an energy difference given in second order perturbation theory by  $\Delta E=2<\chi_0 |V_c|\chi_1>$ . This effect is, in fact, the linear Stark effect for isospin.

One is, of course, with this model not bound to such extreme single particle wave functions but can remain entirely within the shell model. Indeed, the shell model does provide such accidental degeneracy for these 2+ states. Be8 is close to the LS-limit, in which scheme the 2+ and 1+ states arise from the supermultiplets  ${}^{33}P[31]$  for T=1, and  ${}^{13}P[31]$  for T=0. It turns out that <sup>53</sup> these two configurations are degenerate within 100 keV for Kurath's force (0.8 space exchange, 0.2 spin exchange) independent of L/K. As the spin-orbit force is turned on the energy eigenstates change. But Kurath has shown 54 that while the 1<sup>+</sup> states move apart, the 2<sup>+</sup> states remain close together more or less independent of the spin orbit coupling parameter a/K. It should be noted, however, that these energies depend, of course, on the specific force parameters. Hence Cohen and Kurath 10 in a recent consistent treatment of the p-shell do not obtain degenerate 2+ states. However, their overall fit to the Be8 level scheme is not good. The shell model should now be

<sup>&</sup>lt;sup>50</sup> P. Paul, D. Kohler, and K. A. Snover, to be published.

<sup>51</sup> P. Paul, J. B. Thomas, and S. S. Hanna, to appear in Phys. Rev.

<sup>&</sup>lt;sup>52</sup> F. C. Barker, Bull. Am. Phys. Soc. 10, 439 [1965].

<sup>&</sup>lt;sup>53</sup> E. Feenberg and M. Phillips, Phys. Rev. **51**, 597 [1937].

<sup>&</sup>lt;sup>54</sup> D. Kurath, Phys. Rev. 101, 216 [1956].

able to predict not only the strong  $17.6 \rightarrow 16.6$ transition but also the weak branch  $17.6 \rightarrow 16.9$ . We will show how on the basis of the single particle model this can come about in a natural way. Assuming the configuration  $(Li^7 + p) + (Be^7 + n)$  for the 17.6-MeV state (which has T=1), but the split configurations (Li<sup>7</sup>+p) at 16.6 MeV and (Be<sup>7</sup>+n) at 16.9 MeV, the transition is due to the valence particle which is in one case a proton, in the other case a neutron. Each half of the 17.6-MeV state overlaps only with one final state. Using single particle magnetic moments for the M1 operator, one obtains for the various possible single particle transitions the strengths and ratios listed in Table 4. If isospin is conserved for the 1+ states only half of the transition strength listed in Table 4 applies to the transition. Obviously a  $p_{3/2} \rightarrow p_{3/2}$  transition yields about the correct value for the absolute transition strength to the 16.6-MeV state as well as for the ratio of  $17.6 \rightarrow 16.9$  to  $17.6 \rightarrow 16.6$ . However, the near isotropy of the gamma transition to the ground state requires in this simple picture the capture of a p<sub>1/9</sub> particle.

This explains the partial agreement with the Li<sup>7\*</sup> lifetime, which involves a  $p_{1/2} \rightarrow p_{3/2}$  transition. However, such a transition gives only half of the correct strength for the  $17.6 \rightarrow 16.6$  decay and too large a ratio  $(17.6 \rightarrow 16.9)/(17.6 \rightarrow 16.6)$ . One has to increase the proton component in the 17.6-MeV configuration by mixing about 25% of T=0 into this state to obtain the correct ratio, or include some  $p_{3/2} \rightarrow p_{3/2}$  component into the transition. Capture of a p<sub>1/2</sub> proton would lead to channel spin ratio R = (s=2)/(s=1) = 5. For a  $p_{3/2}$  particle R = 0.2. The experimental value is 55 3.2 indicating p<sub>3/2</sub> and p<sub>1/9</sub> mixture. A rigorous shell model calculation requires less than 10% isospin mixing to explain the observed ratio. The proton excess in the 17.6-MeV state is reflected by a neutron excess in the 18.15-MeV state which enhances the transition to the 16.9-MeV state. Indeed at 18.15 MeV the 16.9-MeV transition has greatly increased relative to the 16.6-MeV transition. In this context the fact that the reduced proton width of the 18.15-MeV state is about twofold larger than at 17.6 MeV is somewhat worrysome. However, as explained earlier, it seems plausible that this can be explained by a change in the radial wave function in the sense of a Thomas-Ehrman effect 46.

An experimental discrimination between cluster or shell model description requires an accurate check on the wave functions. In the case of the 17.6-MeV state this was attempted by comparing the radiative widths of the 980-keV state in Li<sup>8</sup> with the theoretical value obtained with the wave functions that give the correct strengths for the transitions  $17.6 \rightarrow 16.6$ and  $17.6 \rightarrow 16.9$  in Be<sup>8</sup>. However, the upper limit for the mean life <sup>12</sup> of  $\tau < 3.5 \times 10^{-14}$  sec in Li<sup>8</sup> is compatible with both the shell model and even a simple core transition. Another parameter very sensitive to the wave function of the state is the channel spin ratio. In the case of the 17.6-MeV state  $R\left(\frac{S=2}{S=1}\right)$  has been found to be <sup>55</sup> 3.2 ± 5. This value can directly be compared to the analogous ratio in the 980-keV state in Li<sup>8</sup>. Experimentally it is obtained from the  $(d p \gamma)$  correlation 11 in the reaction  $Li^{7}(d p \gamma)Li^{8}$ . It turns out that the correlation is not very sensitive to this ratio, but within the large error, the observed ratios in Li8 and Be8 are the same.

To summarize the results concerning the  $2^+$  and  $1^+$  doublets, both the experiments and theoretical calculations lead to the following conclusion: While Marion's model of single particle configurations contains most of the essential features, the shell model (which of course contains the configurations  $(Li^7+p)$  and  $(Li^{7*}+p)$  as major parentages) is capable of explaining the maximal isospin mixing for the  $2^+$  states and the isospin purity of the  $1^+$ 

Barker 52 has done a complete intermediate coupling calculation for Be8 on the basis of maximal isospin splitting for the 2<sup>+</sup> states and obtained good agreement with all gamma transitions and the channel spin ratio at 17.6 MeV. The gamma transitions from  $18.15 \rightarrow 16.6$ , 16.9 are not included. However, his calculation yields only half the observed energy separation for the 1<sup>+</sup> and 2<sup>+</sup> doublets. This calculation was done with a specific radius interpolating the Li<sup>8</sup> and B8 radius. In all probability a somewhat different radial wave function at large radii would lead to the full splitting. This has been proven with a single particle calculation by Dalton and Robson 56 for the 2<sup>+</sup> states. It is of course, well known <sup>46</sup> that oscillator wave functions do not describe the radial dependence correctly at the nuclear surface. This is particularly important for the particle unbound states in Be8.

<sup>&</sup>lt;sup>55</sup> V. Meyer, H. Wuller, H. H. Staub, and R. Zurmuhle, Nucl. Phys. 27, 284 [1961].

<sup>&</sup>lt;sup>56</sup> B. J. Dalton and D. Robson, preprint 1966.

states. It yields good agreement with all available data. A cluster model would require several configurations. In view of the general success of the intermediate coupling shell model in p-shell nuclei it is gratifying that this model does not have to be sacrified in the case of Be<sup>8</sup>.

An estimate of how realistic the single particle description for the Be<sup>8</sup> states is can be obtained from the  $(\text{Li}^7+p)$  and  $(\text{Li}^{7*}+p)$  fractional parentage coefficients in the intermediate coupling model. Such a calculation <sup>10</sup> yields for the sum of the squares of all possible spin combinations: FPC $\approx$ 0.33 for  $J^x=2^+, \approx$ 0.3 for  $J^x=1^+, \approx$ 0.09 for  $J^x=3^+$ . Thus for the  $2^+$  and  $1^+$  states the single particle picture is quite good.

## V. Analog States above 18 MeV

Discrimination between accidental or general breakdown of isospin in Be<sup>8</sup> should not be based on the 2<sup>+</sup>, 1<sup>+</sup> doublets alone but requires an investigation of as many analog states as possible. However, one is limited by the fact that on one side not many more states are known in Li<sup>8</sup>, and on the other side the properties of states in Be<sup>8</sup> in the pertinent energy region have not yet been studied in sufficient detail. Table 5 lists some states in Be<sup>8</sup> which come into consideration as analog states to the states in Li<sup>8</sup> at 2.26, 3.21 and 5.0 MeV.

$_{\rm MeV}^{\rm energy}$	$J^{\pi}$	$\Gamma({ m keV})$	$\frac{\gamma_{\rm n}^2}{\gamma_{\rm p}^2}$	$\frac{\gamma n^{2*}}{\gamma p^{2*}}$
18.9	2-	> 505	5	
19.05	$\stackrel{2^-}{\leqq}_{3^+}$	270	1/13	
19.22	$\overline{3^+}$	190	5.2	
20.36	1-	1160	5.2	>1
21.5	3(+)	1000	1	
22.2	?	1000		<1
23.6	$(1^-, 2^-)$	5000		

Table 5. Properties of some highly excited states in Be<sup>8</sup>. Most of these states show excess of one single particle charge state over the other.

The 3<sup>+</sup> state in Li<sup>8</sup> at 2.26 MeV has been thoroughly studied. In Be<sup>8</sup> the pair of states at 19.05 and 19.22 MeV have been considered as another analog doublet, with  $J=3^+$ . The reason for assuming such a doublet lies with the fact that the well known 3<sup>+</sup> state at 19.22 MeV has a large excess of neutron reduced width. A reduced width ratio  $\gamma_n^2/\gamma_p^2 \approx 5$  has been obtained from the cross sections of Li<sup>7</sup>(p, n)<sup>27</sup> and Li<sup>7</sup>(p, p)<sup>26</sup>. The values were calcu-

lated from the partial widths using the standard R-matrix theory penetration factor  $P = \varrho_2/A_1$  for p-waves. A calculation using a realistic nuclear potential probably changes the penetration factors somewhat and might bring the neutron and proton reduced width into better agreement. The COULOMB barrier for protons bombarding Li7 amounts to about 1.2 MeV, so that protons from this state have an excess of 1 MeV over the barrier energy. One would, therefore, not assume Coulomb effects to be great importance. If one explains this ratio with the single particle picture and isospin mixture, an isospin admixture of 30% (amplitude square) is required. To achieve this degree of admixture the mixing state cannot lie far away, in particular since states with J=3 are not expected to have a large width. The 19.22-MeV state has in fact only  $\Gamma = 190$  keV. The 19.05-MeV state  $\Gamma \approx 270 \text{ keV}$  fulfills these requirements and has a large proton excess 26 with  $\gamma_{\rm p}^2/\gamma_{\rm n}^2 \approx 13$ . It may very well be the case that unbound states only have to be close enough to each other to have overlapping widths to allow Coulomb mixing. Such calculation using unbound states have not yet been performed but it is plausible that the COULOMB mixing for states with similar wave functions is proportional to the energy overlap of both states. In this region the states are, so to say, degenerate, and the weak Coulomb force can induce transitions between the two. The reactions 38  $Be^{9}(He^{3}, \alpha)$  and  $Be^{9}(d, t)$  populate the 2.26 MeVstate in Li8, and some state(s) in Be8 around 19 MeV. The J=3 state at 21.5 MeV is too far away, and appears to have <sup>28</sup>  $\gamma_n^2 \approx \gamma_p^2$ . Barker <sup>52</sup> has, therefore, assumed in his calculations the 19.05-MeV and 19.22-MeV states to be a doublet in the same sense as the 1<sup>+</sup> and 2<sup>+</sup> states, with a degree of mixing, and seems to be able to incorporate it naturally into the intermediate coupling model. It should be noted that the 3<sup>+</sup> states, in LS-coupling, do not arise from the nearly degenerate 33P[31] and 13P[31] supermultiplets, but from <sup>33</sup>D[31] and <sup>13</sup>D[31] which differ by about 1 MeV for a Kurath force. Apparently the spin-orbit force is able to move the states close enough together to achieve some isospin mixing. It is interesting to note that in Kurath's recent calculation the 3<sup>+</sup> states lie actually closer together than the 2<sup>+</sup> states. The existence of these two levels with the complementary reduced width ratios inspires some confidence that the neutron or proton excess does indeed indicate isospin mixing in this energy range. The picture is, however, rather speculative as long as the radiative decay properties of these states are not better known.

A discussion of analog states to the next higher states in Li<sup>8</sup> stands on even less firm grounds since the spins of these states are not even known uniquely. There is, however, some evidence (with the same qualifications as mentioned above) that there exist more states with some isospin impurity above 18 MeV. One well known case is the 2 state at 18.9 MeV. This state also shows a ratio 25 of  $\gamma_n^2/\gamma_p^2 \approx 5$ . It is a curious fact that there happens to be a tentative state with 2 in Li8, the analog of which would, however, be expected around 22 MeV in Be8. There is a possible 2 state proposed at 23.6 MeV observed  $^{57}$  in Li<sup>7</sup>(p,  $\gamma_1$ ). The average energy between this and the 18.9-MeV state is 21.3 MeV which is rather close to the energy of 22 MeV where the analog state could be expected. It remains an open question whether two broad states, separated by 4.7 MeV, could be mixed significantly by Coulomb forces. However, such pairing would

nicely explain the large neutron excess in the 18.9-MeV level.

A very direct way of measuring the ratio of neutron to proton reduced widths of states in this region is offered by a simultaneous study of Li<sup>7</sup>(p, n') and Li<sup>7</sup>(p, p') through the gamma decay of the final mirror states Li7\* and Be7\*. Since these gamma transitions are isotropic one obtains directly the ratio of total cross sections. Fig. 4 shows the excitation 60 function of the ratio  $\sigma(p, n')/\sigma(p, p')$  from 3 to 7 MeV. In the smooth region of this curve the ratio is expected to be smaller than 1 because the reaction allows more direct ways to the proton channel than to the neutron channel. Rapidly energy dependent deviations from the average that coincide with resonances in the individual cross sections are indicative of either neutron or proton excess in this resonance, with due allowance for differences in the respective Q-values and penetration factors. The curve shows a neutron excess in a resonance near 3.3 MeV proton energy and a proton excess at about 5.5 MeV. The lower anomaly is most likely identical

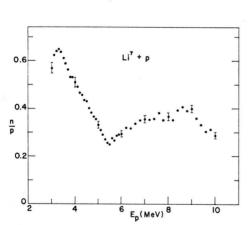


Fig. 4. Excitation function of the ratio  $\sigma(p,n')/\sigma(p,p')$  in the bombardment of Li<sup>7</sup> with protons from 3 MeV to 7 MeV bombarding energy. The ratio is obtained directly by comparing the 480-keV and 432-keV gamma rays from Li<sup>7\*</sup> and Be<sup>7\*</sup> in the same spectrum <sup>60</sup>. Strong fluctuations of the ratio indicate a change of the  $\gamma_n^2/\gamma_p^2$  ratio.

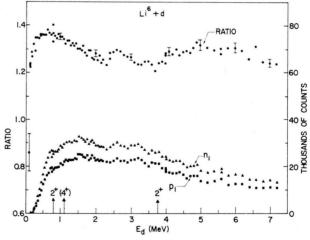


Fig. 5. Cross section ratio  $\sigma(d,n')/\sigma(d,p')$  of the reactions  $\operatorname{Li}^6(d,n')\operatorname{Be}^{7*}$  and  $\operatorname{Li}^6(d,p')\operatorname{Li}^{7*}$ . The ratio is obtained by simultaneously measuring the isotropic gamma rays from both final states. Multiplication with 0.82 corrects for differences in detector efficiency. The average value is in very good agreement with stripping theory predictions  $^{61}$ . The individual cross sections do not show resonance structure.

<sup>8</sup> W. Greiner, Nucl. Phys. 49, 522 [1963].

<sup>61</sup> S. M. Austin, P. Paul, A. Cheung, S. S. Hanna, and W. E. Meyerhof, Bull. Am. Phys. Soc. 11, 10 [1966].

<sup>57</sup> I.V. MITCHELL and R.B. TAYLOB, Nucl. Phys. 44, 664 [1963]. This state is considered as a giant dipole state based on the 2.9-MeV state. It has been shown in O<sup>16</sup> by Greiner <sup>58</sup> that large isospin mixing can occur in giant dipole states. A similar case has recently been considered <sup>59</sup> in C<sup>12</sup>.

<sup>&</sup>lt;sup>59</sup> R. E. SEGEL, S. S. HANNA, and R. G. Allas, Phys. Rev. 139, B 818 [1965].

<sup>&</sup>lt;sup>60</sup> S. S. HANNA, P. PAUL, S. M. AUSTIN, and W. E. MEYERHOF, Bull. Am. Phys. Soc. 11, 10 [1966].

with the  $1^-$  state observed in the same reaction at 3.5 MeV with a width of 1.3 MeV by Buccino et al.  $^{29}.$  They obtain for the reduced widths  $\gamma_{\rm n}{}^2/\gamma_{\rm p}{}^2\!\approx\!5$  which would again be explained by an isospin mixture of 30%. The anomaly at 5.5 MeV coincides with a resonance seen by Gleyvod et al.  $^{62}$  in inelastic proton scattering. This resonance does not appear in elastic proton scattering. The spin is as yet unknown but one is tempted to match the 20.35- and 22.0-MeV resonances as the analog pair to the  $1^\pm$  state at 3.21 MeV in Li8. But clearly much more experimental work will be needed in both Li8 and Be8, and a more realistic theoretical evaluation of penetration factors is required before these speculative pairs can be taken seriously.

In the region above 20 MeV isospin mixing is expected to be small according to the argument put forward by Wilkinson 63, i. e. that reactions in which a great many channels are open for decay proceed so fast that there is no time for the COULOMB force to mix different isospin states. The channel (Li<sup>6</sup> + d) offers access to the region of excitation beyond 22 MeV. The total cross sections of the reactions Li<sup>7</sup>(d, n')Be<sup>7</sup> and Li<sup>6</sup>(d, p')Li<sup>7</sup> can again be measured 61 simultaneously. If the reactions proceed by the stripping mechanism, the ratio is a check on the charge symmetry of the final states. If resonances are observed, the ratio measures again the relative size of neutron to proton reduced widths of the associated compound states. The experimental cross section ratio as a function of energy, covering Be8 from 22.5 MeV to 28.4 MeV is shown in Fig. 5. It has, above 1 MeV bombarding energy, a rather smooth dependence. After accounting for differences in detection efficiency one obtains an average value of 1.15 which is in marked agreement with the expectations for a stripping reaction leading to charge symmetric final states 61. Since the individual cross sections do not show definite resonances, nothing can be said about the isospin character of states at these excitation energies. The small wiggles appearing in the curve might represent small energy dependent isospin mixing ratios in the background.

#### VI. Conclusion

All experimental evidence establishes isospin mixing in the two 2<sup>+</sup> states at 16.6 MeV and 16.9 MeV that must be almost maximal. Neither of the two is

therefore strictly the analog state of the Li<sup>8</sup> ground state. However, the first excited state in Li<sup>8</sup> has a rather pure analog state in Be<sup>8</sup> at 17.6 MeV. The 1<sup>+</sup> pair of states at 17.6 MeV and 18.15 MeV mixes by at most 10%. The analog state of the 3+ state at 2.26 in Li<sup>8</sup> is very likely concentrated in the state at 19.05 MeV in Be8. There is some evidence that this state might mix to about 30% with the state at 19.22 MeV. The broad states at 3.21 MeV and 5 MeV in Li<sup>8</sup> might have analog states in Be<sup>8</sup> which contain as much as 30% of isospin T = 0. In the first case the analog pair could be the states at 20.3 MeV and 22 MeV with  $J=1^{-}$ . The second required pair could correspond to the states at 18.9 MeV and 23.6 MeV with  $J^{\pi} = 2^{-}$ . Such a pairing off could explain all cases in which a large excess of proton or neutron reduced width has been found in Be<sup>8</sup>.

It is a peculiar characteristic that at least the lower states, and to a lesser degree, all states involved have a strong single particle character. The fact of large isospin admixture in the 16.6-MeV and 16.9-MeV states is, of course, not tied to this particular configuration. But it is a nice point in favor of a cluster model description of Be8. Such model favors the cluster configuration of channels with thresholds close to the level energy. In the region of the final T=1 states these are the single particle channels. The next more complicated clusters which can form T=1, are  $(\mathrm{He^5}+\mathrm{He^3})$  and  $(\mathrm{Li^5}+\mathrm{t})$  which correspond to open channels only above 21 MeV. It is therefore, below 20 MeV, quite reasonable to expect the T=1 states to have large reduced neutron and proton widths. The experimental evidence, however, does not support the assumption of any simple cluster configurations except possibly for the 2<sup>+</sup> pair. Instead there seems to be good reason to assume that the intermediate coupling shell model is the correct way to describe Be8 in the region of excitation above 16 MeV. The exceptionally large isospin mixing is contained in this model as an unlikely accident. In this sense, Be8 seems to be a normal, if somewhat ill-behaved, member of the nuclear p-shell.

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<sup>&</sup>lt;sup>62</sup> R. GLEYVOD, N. P. HEYDENBURG, and I. M. NAQIB, Nucl. Phys. **63**, 650 [1965].

<sup>63</sup> D. H. Wilkinson, Phil. Mag. 2, 83 [1957].